

ECE 476

SPRING 2010

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# ARITHMETIC CIRCUITS 05 March 2010

## INTEGERS

Always positive numbers = UNSIGNED

Numbers can be negative also = SIGNED

## Decimal numbers

radix = base = 10

$$D = d_{n-1} d_{n-2} \dots d_0$$

DIGITS (n-tuple number)

$$V(D) = \dots + d_{n-1} \times 10^n + d_{n-2} \times 10^{n-2} + \dots + d_1 \times 10^1 + d_0 \times 10^0$$

Value of the number

## BINARY NUMBERS

radix = base = 2

$$B = b_{n-1} b_{n-2} b_{n-3} \dots b_1 b_0$$

$$V(B) = b_{n-1} \times 2^{n-1} + b_{n-2} \times 2^{n-2} + \dots + b_1 \times 2^1 + b_0 \times 2^0$$

$$V(B) = \sum_{i=0}^{n-1} b_i \times 2^i$$

$$\begin{array}{c} (11011)_2 \\ \begin{array}{c} | \quad | \quad | \quad | \quad | \\ 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \end{array} \end{array} \Rightarrow 2^4 + 2^3 + 0 \times 2^2 + 2^1 + 2^0 = 16 + 8 + 0 + 2 + 1 = 27_{10}$$

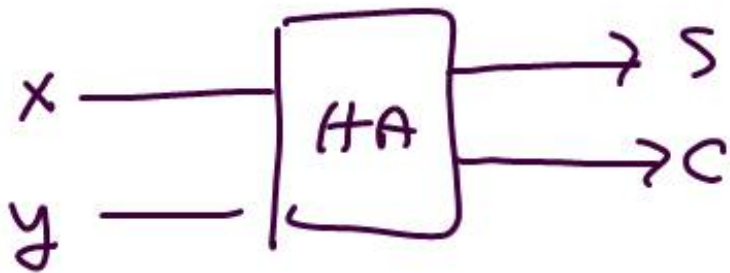
Octal (radix 8)

$$(100101101100)_2 = (?)_8 = 4554_8$$

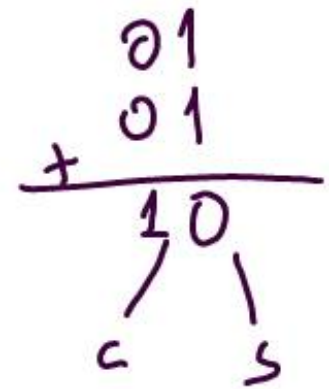
$$\text{Hexadecimal (radix 16)} = 96C_{16}$$

# ADDITION of UNSIGNED NUMBERS

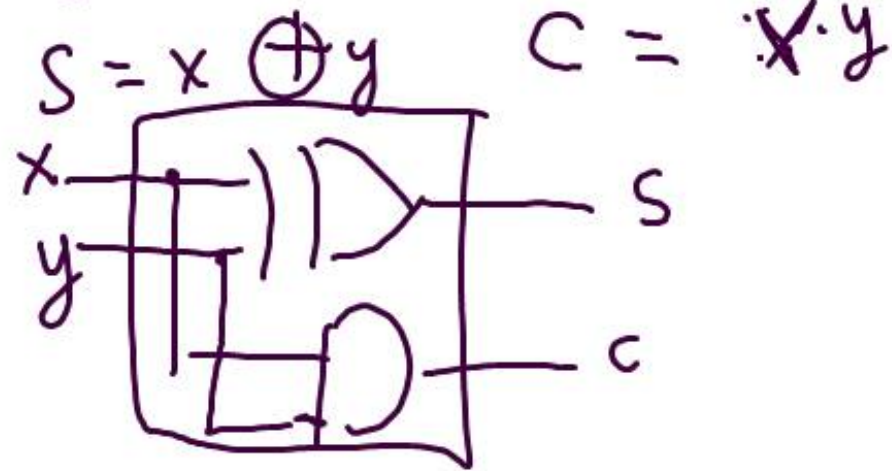
HALF ADDER



x	y	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



FULL ADDER



# FULL ADDER

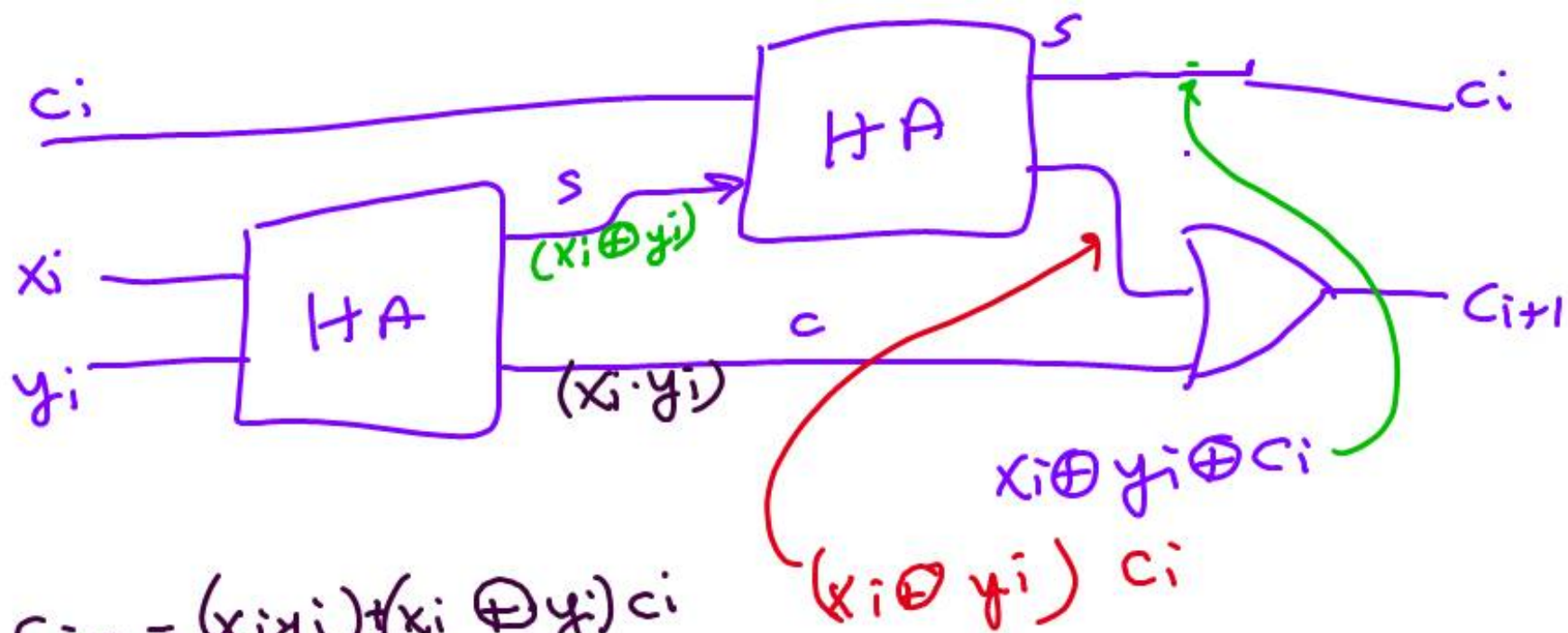
$C_i$	$x_i$	$y_i$	$S_i$	$C_{i+1}$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$x_i y_i$	00	01	11	10
$C_i$	0	1	0	1
$\neg$	1	0	1	0

$$S_i = x_i \oplus y_i \oplus C_i$$

$C_i$	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$C_{i+1} = x_i y_i + x_i C_i + y_i C_i$$



$$c_{i+1} = (x_i y_i) + (x_i \oplus y_i) c_i$$

$$= x_i y_i + (\bar{x}_i y_i + x_i \bar{y}_i) c_i$$

$$= x_i y_i + c_i \bar{x}_i y_i + c_i x_i \bar{y}_i$$

$$= y_i (x_i + c_i \bar{x}_i) + c_i x_i \bar{y}_i$$

$$= y_i (x_i + c_i) \underbrace{(x_i + \bar{x}_i)}_1$$

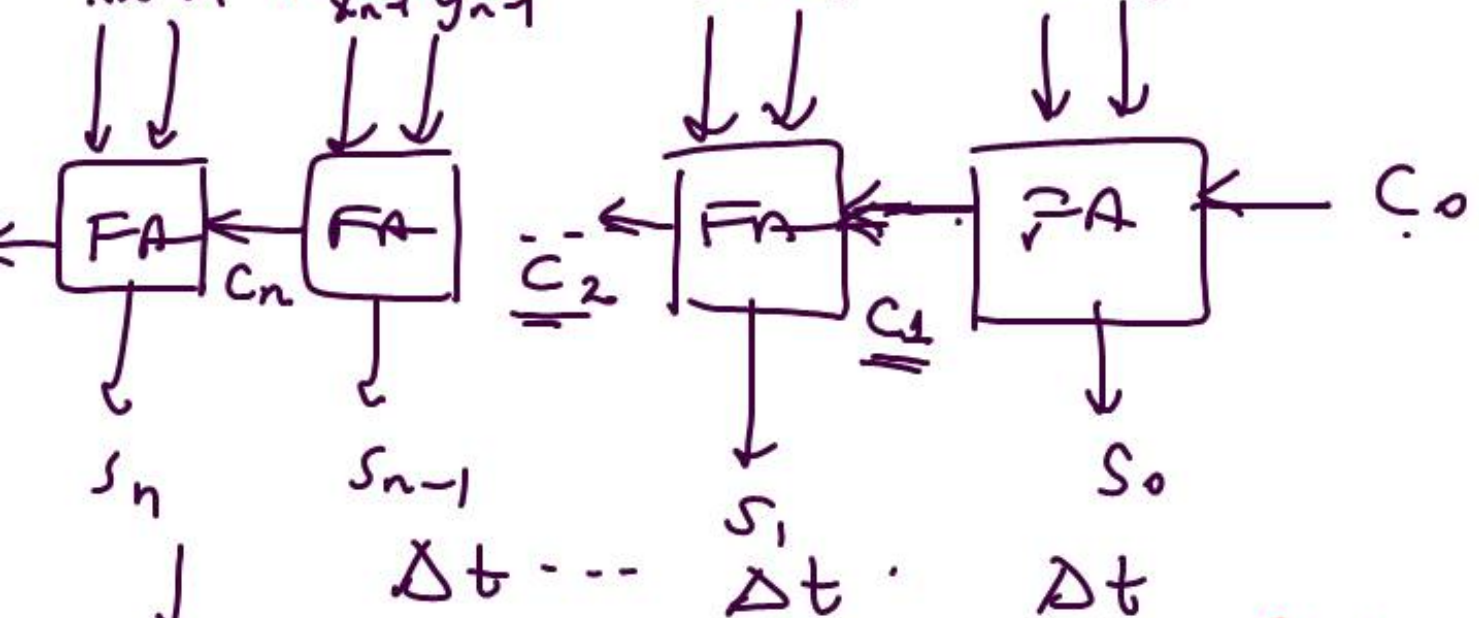
$$= y_i x_i + y_i c_i + c_i x_i y_i$$

$$= y_i x_i + c_i (y_i + x_i y_i)$$

$$= y_i x_i + c_i (y_i + x_i) (y_i + \bar{y}_i)$$

$$= y_i x_i + c_i y_i + c_i x_i$$

# RIPPLE CARRY ADDER

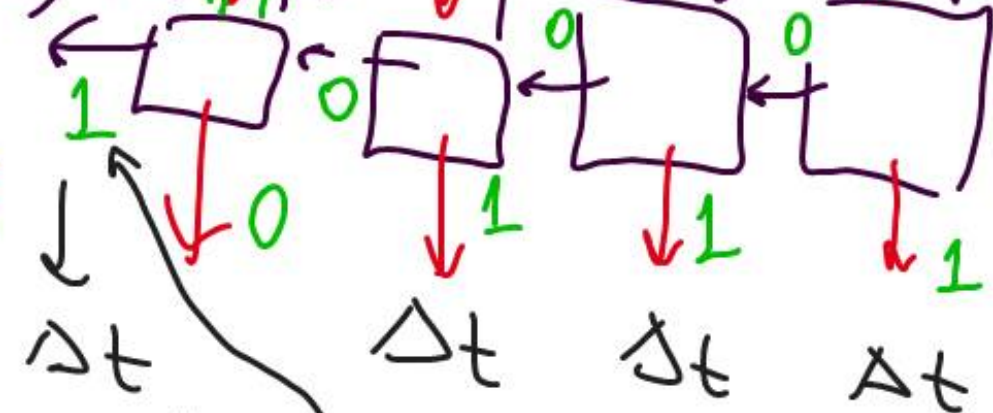


$\frac{n \cdot \Delta t}{\text{Ripple}}$

need to be waited.

$$\begin{array}{r} 1100 = 12 \\ 1011 = 11 \\ \hline 23 \end{array}$$

$$(1200) + (1011) =$$

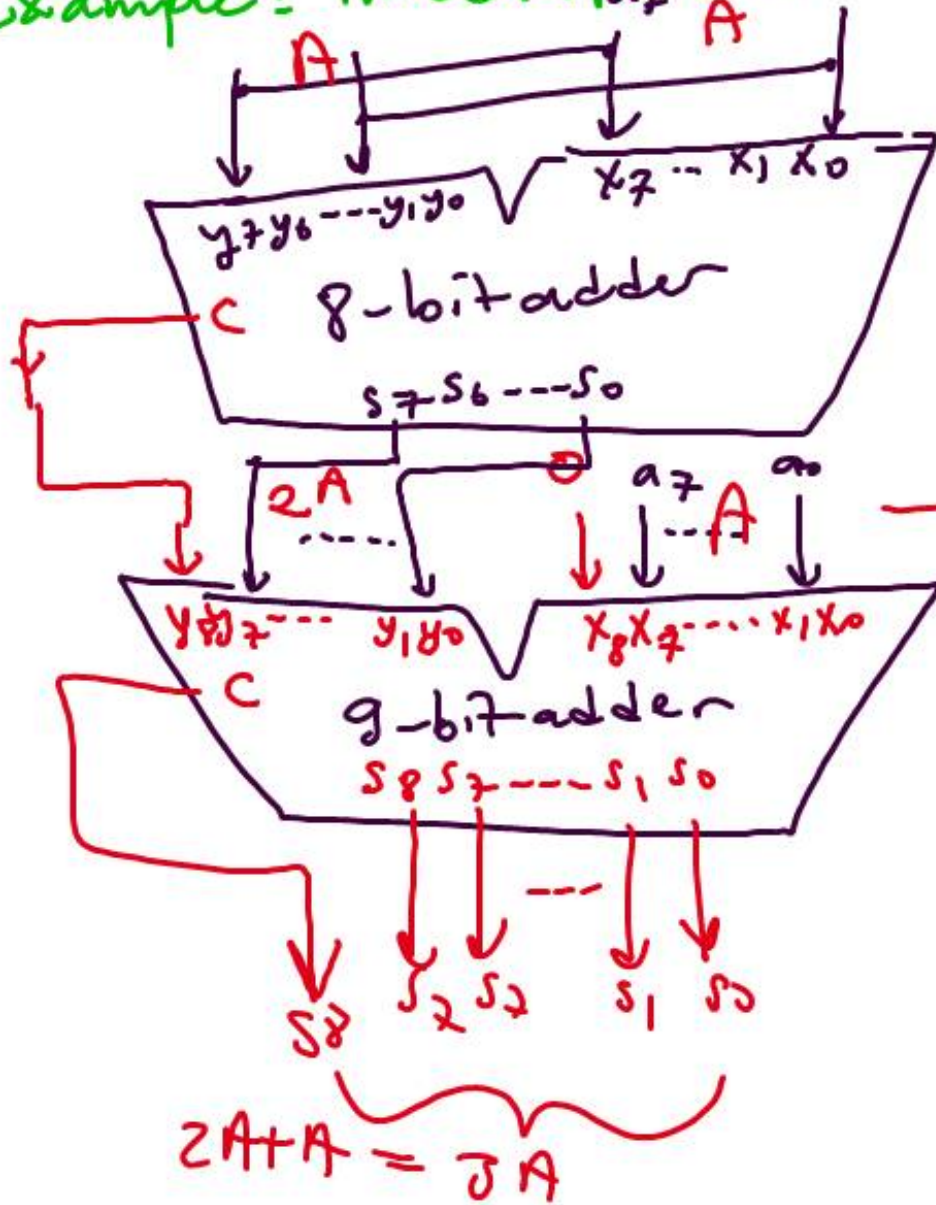


$$\begin{array}{r} = 1011 \\ 16 + 4 + 2 + 1 \\ = 23 \end{array}$$

Wait =  $4 \times \Delta t$  to obtain

Example: Multiplier

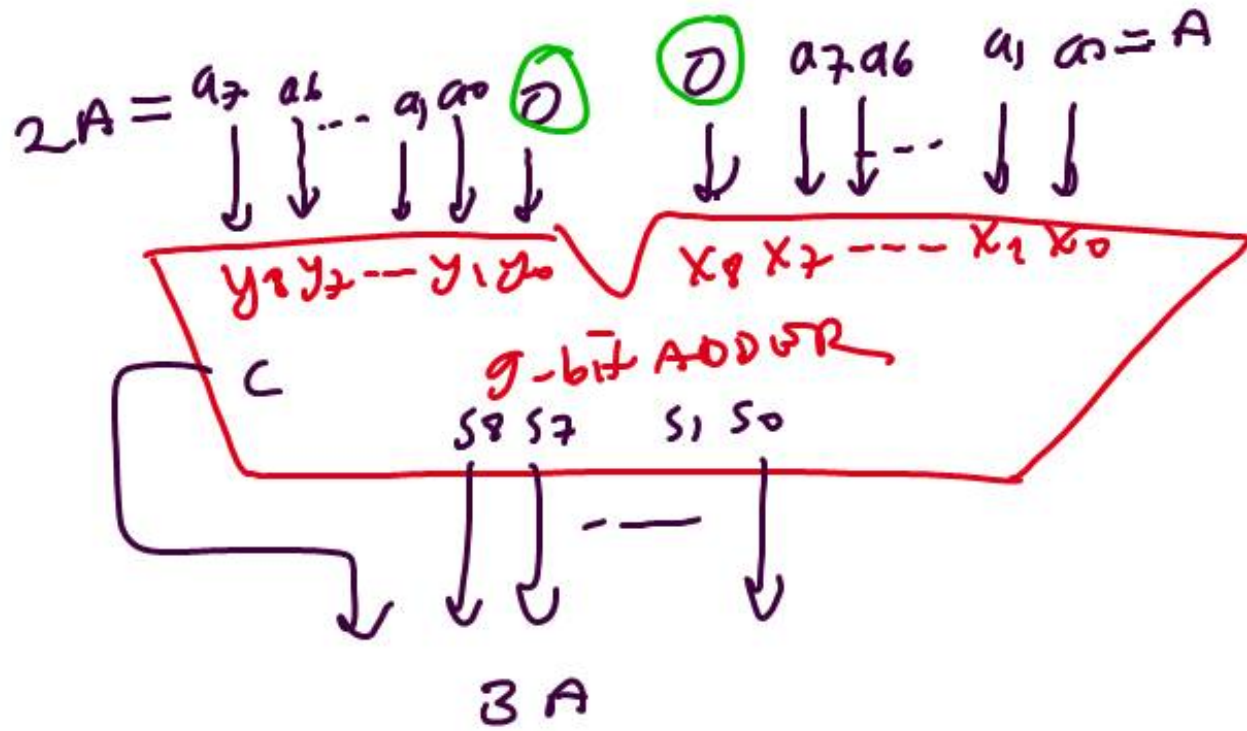
$$A = a_7 a_6 \dots a_1 a_0$$



$\rightarrow 0A = A$

$A \times 3 = 3A$   
 multiplication by  
 summation





- $0011 = 3_{10}$
- $00110 = 6_{10}$
- $01100 = 12_{10}$

$= a_7 a_6 a_5 \dots a_1 a_0 0$  } = Multiply by 2  
 shifting 1 bit left.  
 and adding 0 from right

# SIGNED NUMBERS

1) Sign and magnitude

MSB  $\Rightarrow$  sign

$$\begin{array}{r} 0101 \\ \downarrow \quad \checkmark \\ +5 = +5 \end{array}$$

$$\begin{array}{r} 1101 \\ \underline{-5} \\ -5 \end{array}$$

2) One's complement

$$\begin{array}{l} 0111 = +7_{10} \longrightarrow 1000 = -7 \\ +1111 \rightarrow 0000 \\ = -0 \quad +0 \end{array}$$

Problem! 2 zeros.

3) 2's complement  
 $0000 \rightarrow$  one zero!

$$\begin{array}{r} \overline{0101} \\ +5 \end{array}$$

$$\Rightarrow \text{2's comp: } \begin{array}{r} 1010 \\ + \quad 1 \\ \hline 1011 \\ = -5 \end{array}$$

4-bit numbers:

$$1000 = \begin{array}{r} 0111 \\ \underline{1000} \\ -8 \end{array}$$

0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111

$1101 \rightarrow$

$$\begin{array}{r} 0010 \\ \underline{0011} \\ -3 \end{array}$$

$(2^{n-1} - 1)$

$1001 \rightarrow$

$$\begin{array}{r} 0110 \\ \underline{0111} \\ -2 \end{array}$$

-8	← 1000
-7	← 1001
-6	← 1010
-5	← 1011
-4	← 1100
-3	← 1101
-2	← 1110
-1	← 1111

$(-2^{n-1})$

$n=4$

$2^{4-1} - 1 = 8 - 1 = 7$

$-2^{n-1} = -2^3 = -8$

# OVERFLOW

10 Mar 2010

In 2's complement numbers overflow may occur (in addition + subtraction)

Signs same!

Signs same!

$$\begin{array}{r} +4 \\ +5 \\ \hline +9 \end{array}$$

$$\begin{array}{r} -6 \\ -5 \\ \hline -11 \end{array}$$

$$\begin{array}{r} +5 \\ + -2 \\ \hline +3 \end{array}$$

$$\begin{array}{r} -5 \\ +4 \\ \hline -1 \end{array}$$

Cannot be represented by 4 bits!  $\Rightarrow$  OVERFLOW

4 bits to represent:  $\left\{ \begin{array}{l} 0, 1, 2, 3, 4, 5, 6, 7 \\ -1, -2, -3, -4, -5, -6, -7, -8 \end{array} \right\}$   
in 2's complement representation

n-bits  $\rightarrow$   $2^{n-1} - 1$  positive,  $2^{n-1}$  negative numbers  
VALUE VALUE

$$\begin{array}{r}
 +4 \\
 +5 \\
 \hline
 +9 \\
 \hline
 \end{array}
 \begin{array}{r}
 0100 \\
 +0101 \\
 \hline
 1001 \\
 \hline
 \end{array}$$

$(-7) \times$   
 $c_1 = 0$   
 $c_2 = 0$   
 $c_3 = 1$   
 $c_4 = 0$

Have different values!  
 OVERFLOW!  
 $overflow = c_3 \oplus c_4$

$$\begin{array}{r}
 -6 \\
 + -5 \\
 \hline
 -11 \\
 \hline
 \end{array}
 \begin{array}{r}
 1010 \\
 +1011 \\
 \hline
 10101 \\
 \hline
 \end{array}$$

$-6 = 0110$   
 $1001$   
 $\hline$   
 $1010$

$c_3 = 0$   
 $c_4 = 1$

$overflow = c_3 \oplus c_4 = 1$

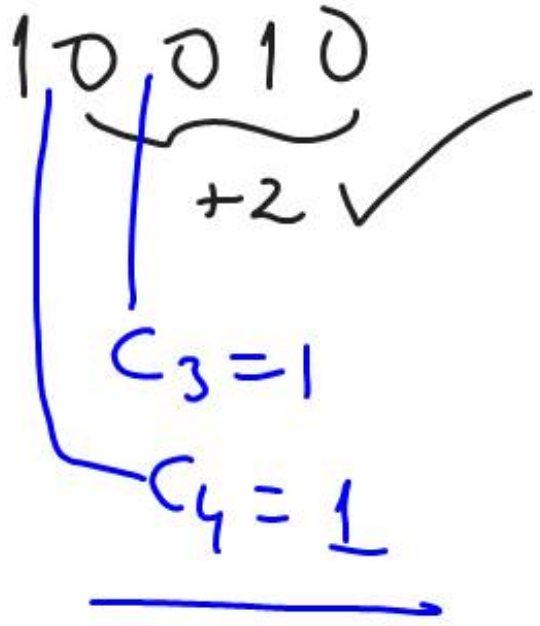
$-5 \Rightarrow 1011$

$$\begin{array}{r}
 0101 \\
 +1010 \\
 \hline
 1011
 \end{array}$$

$$\begin{array}{r}
 +5 \\
 + -3 \\
 \hline
 +2
 \end{array}$$

$$\begin{array}{r}
 0101 \\
 + 1101 \\
 \hline
 10010
 \end{array}$$

$$\begin{array}{r}
 -3 = 0011 \\
 1100 \\
 \hline
 1101
 \end{array}$$



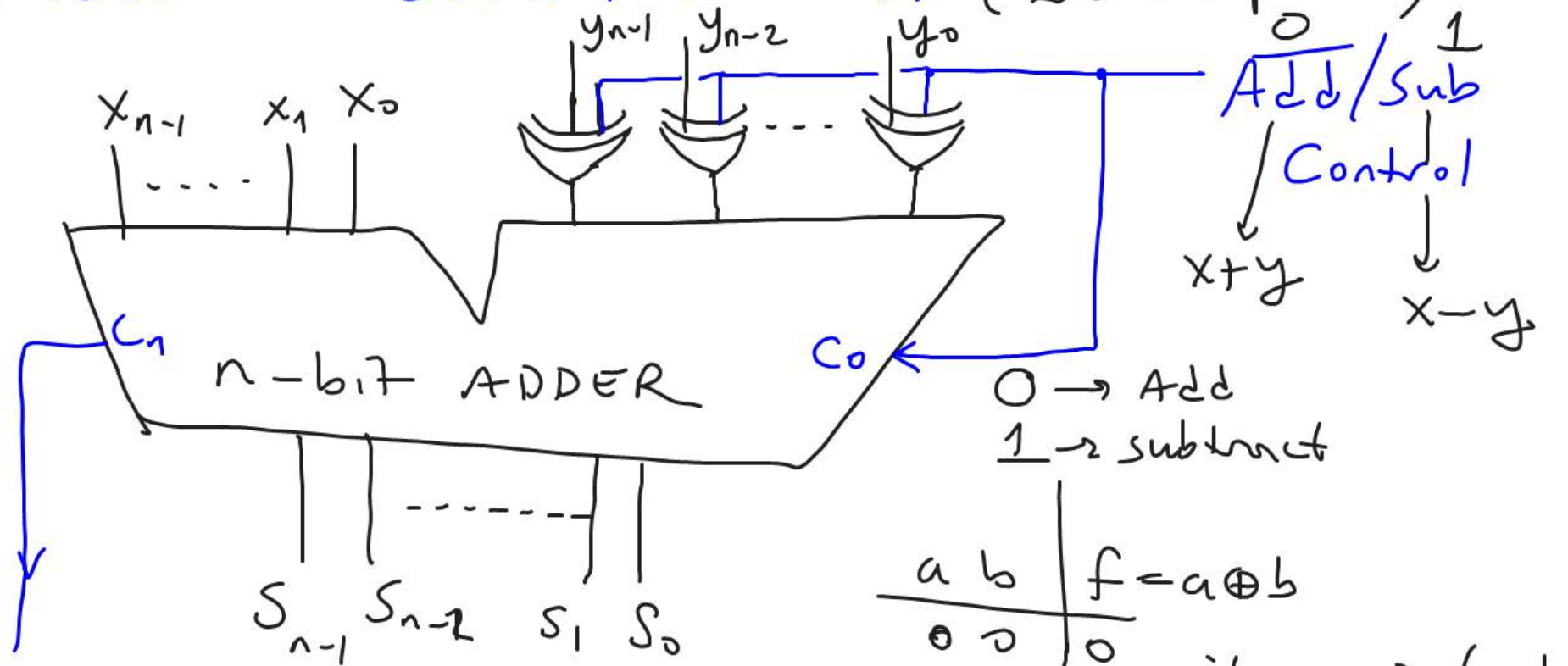
$C_4 \oplus C_3 = 0$   
 No overflow ✓

In General:

For n-bits 2's complement  
 addition, subtraction  
 overflow is detected  
 by:

$$V = C_n \oplus C_{n-1}$$

# ADDER & SUBTRACTOR UNIT (2's complement)



$$X + (-y) = X - y$$

a	b	$f = a \oplus b$
0	0	0
0	1	1
1	0	1
1	1	0

if  $a=0 \Rightarrow f=b$   
 if  $a=1 \Rightarrow f=\bar{b}$

# FAST ADDERS (Look Ahead Carry Adder)

$$C_{i+1} = x_i y_i + x_i c_i + y_i c_i$$

$$C_{i+1} = x_i y_i + (x_i + y_i) c_i$$

$$C_{i+1} = g_i + p_i \cdot c_i$$

$$C_i = g_{i-1} + p_{i-1} c_{i-1}$$

$g_i$  = generation function =  $x_i y_i$

$p_i$  = propagation function =  $x_i + y_i$

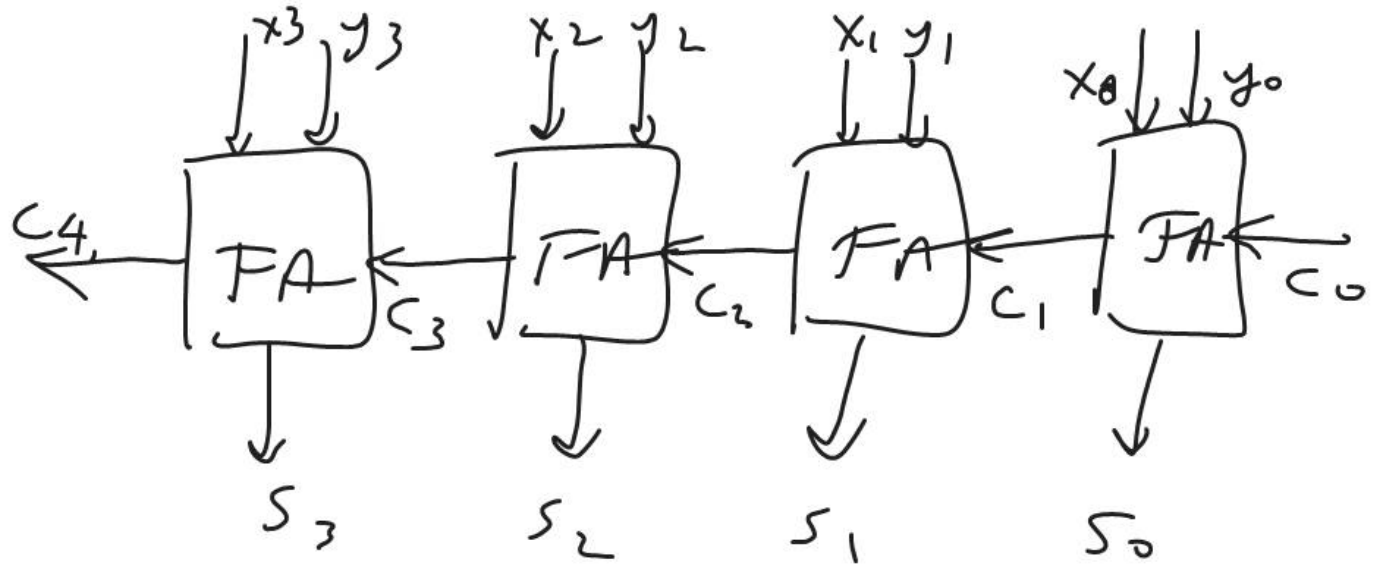
$$C_{i+1} = g_i + p_i (g_{i-1} + p_{i-1} c_{i-1})$$

$$C_{i+1} = g_i + p_i g_{i-1} + p_i p_{i-1} c_{i-1}$$
$$= g_i + p_i g_{i-1} + p_i p_{i-1} (g_{i-2} + p_{i-2} c_{i-2}) =$$

$$\begin{aligned} &= g_i + p_i g_{i-1} + \\ & p_i p_{i-1} g_{i-2} + \\ & p_i p_{i-1} p_{i-2} c_{i-2} \end{aligned}$$



# 4-bit Adder



$$C_4 = \underbrace{x_3 y_3}_{g_3} + \underbrace{(x_3 + y_3)}_{p_3} \cdot C_3$$

$$C_4 = g_3 + p_3 C_3$$

$$C_4 = g_3 + p_3 (g_2 + p_2 C_2)$$

$$C_4 = g_3 + p_3 g_2 + p_3 p_2 C_2$$

$$= g_3 + p_3 g_2 + p_3 p_2 (g_1 + p_1 C_1)$$

$$= g_3 + p_3 g_2 + p_3 p_2 g_1 + p_3 p_2 C_1$$

$$= g_3 + p_3 g_2 + p_3 p_2 g_1 + p_3 p_2 p_1 (g_0 + p_0 C_0)$$

$$C_4 = g_3 + p_3 g_2 + p_3 p_2 g_1 + p_3 p_2 p_1 g_0 + p_3 p_2 p_1 C_0$$

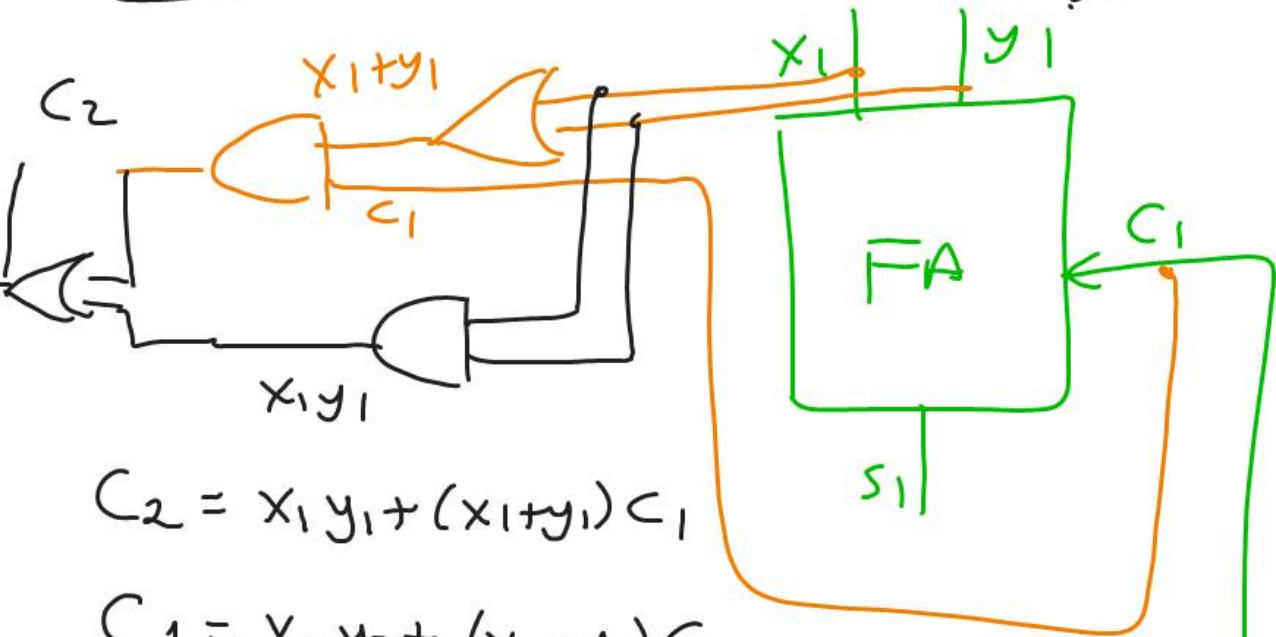
$$g_3 = x_3 y_3 \quad p_3 = (x_3 + y_3)$$

$$g_2 = x_2 y_2 \quad p_2 = (x_2 + y_2)$$

$$g_1 = x_1 y_1 \quad p_1 = (x_1 + y_1)$$

$$g_0 = x_0 y_0 \quad p_0 = (x_0 + y_0)$$

ex 2-bit Fast Adder  $x_0, y_0$

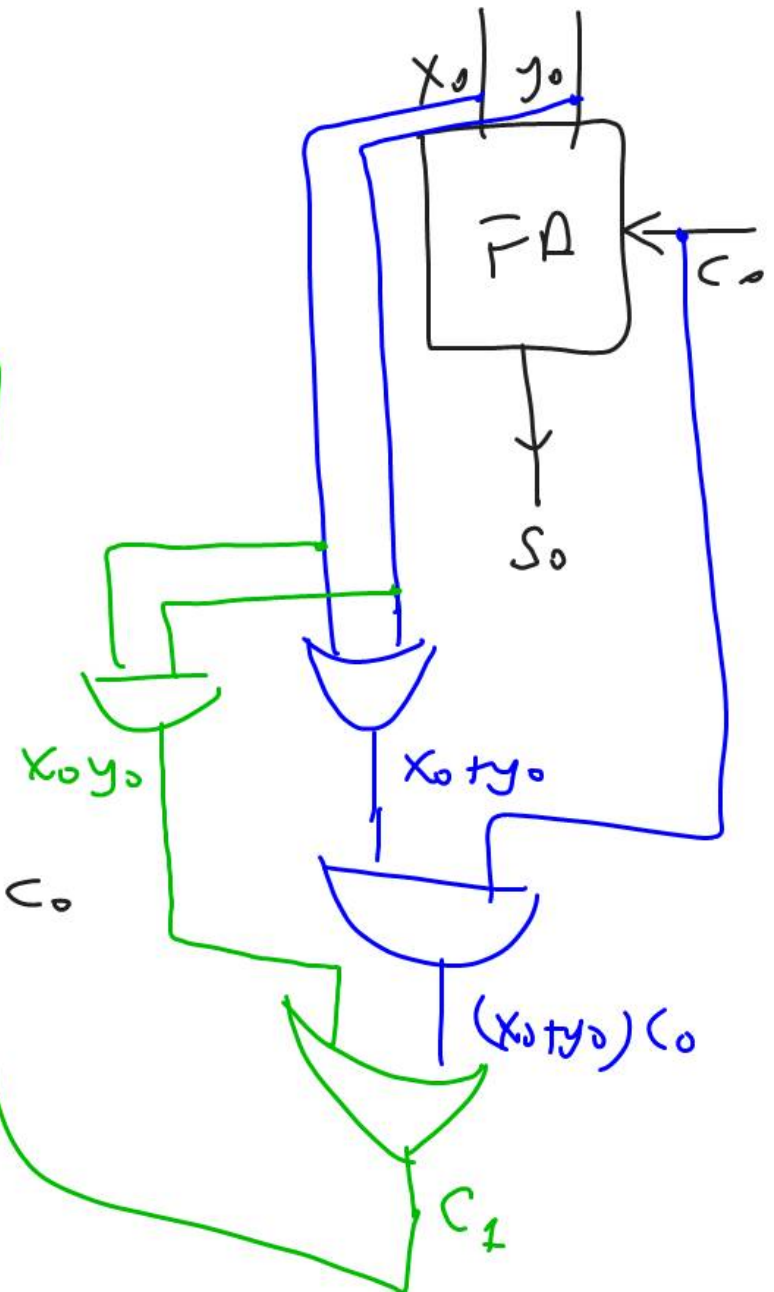


$$C_2 = x_1 y_1 + (x_1 + y_1) C_1$$

$$C_1 = x_0 y_0 + (x_0 + y_0) C_0$$

$$C_2 = x_1 y_1 + (x_1 + y_1)(x_0 y_0) + (x_1 + y_1)(x_0 + y_0) C_0$$

$$C_1 = x_0 y_0 + (x_0 + y_0) C_0$$



2 hours in INT-3 Classroom,

11 Mar 2020

→ Design of Arithmetic circuit

→ Multibit signals

→ Arithmetic Assignment statements

$n$ -bit full adder → unsigned numbers

→ Multiplications

→ signed numbers

16-bit FA

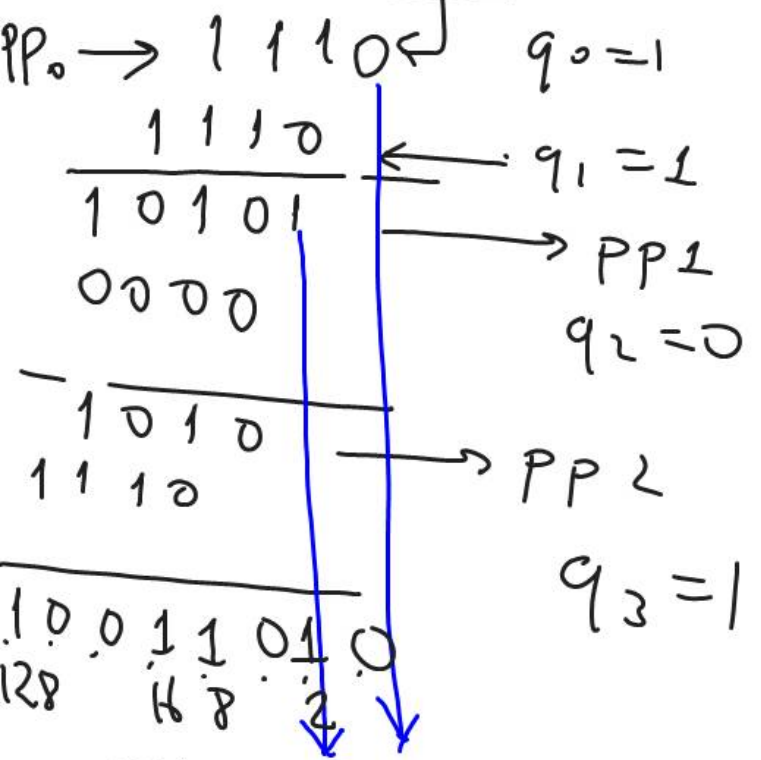
17 Mar 2020  
©

M 1110  
Qx 1011

M = m<sub>3</sub> m<sub>2</sub> m<sub>1</sub> m<sub>0</sub>  
Q = q<sub>3</sub> q<sub>2</sub> q<sub>1</sub> q<sub>0</sub>

$$P = M \times Q \text{ (4-bit)}$$

$$P = P_7 P_6 P_5 P_4 P_3 P_2 P_1 P_0 \text{ (8-bit)}$$

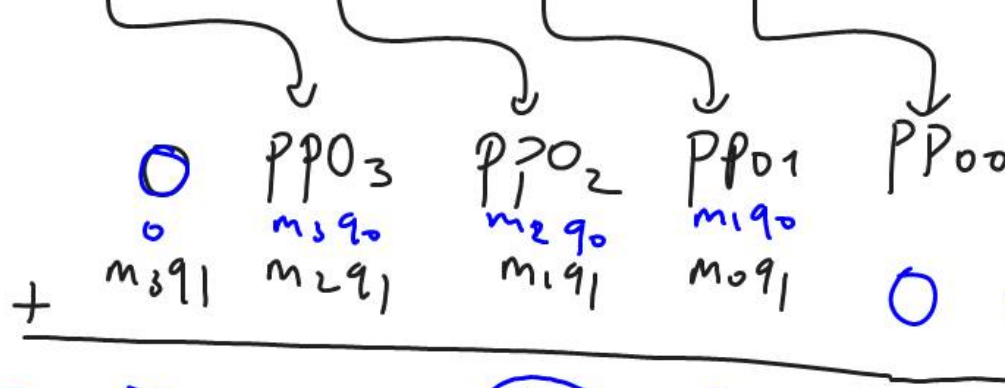


128 64 32 16 8 4 2  
154<sub>10</sub>

# ARRAY MULTIPLIER

$$PP0 = M \text{ AND } Q$$

$$= \underbrace{m_3 q_0}_{PP0_3} \quad \underbrace{m_2 q_0}_{PP0_2} \quad \underbrace{m_1 q_0}_{PP0_1} \quad \underbrace{m_0 q_0}_{PP0_0} \quad M \text{ AND } q_0$$

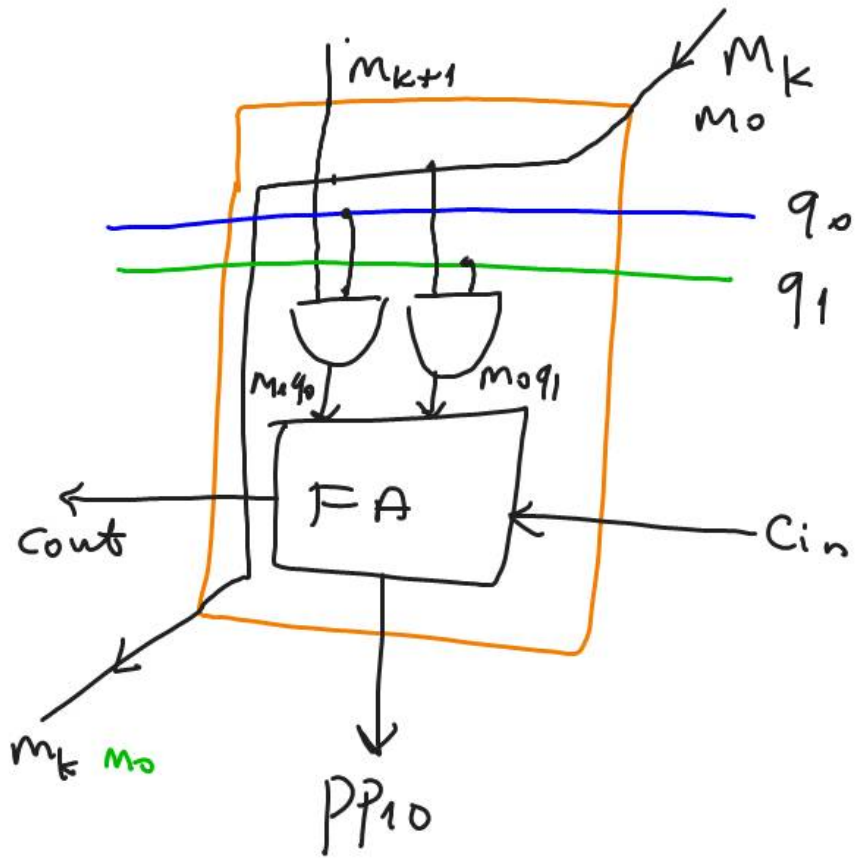


M AND  $q_1$   
and shift 1 bit  
LEFT

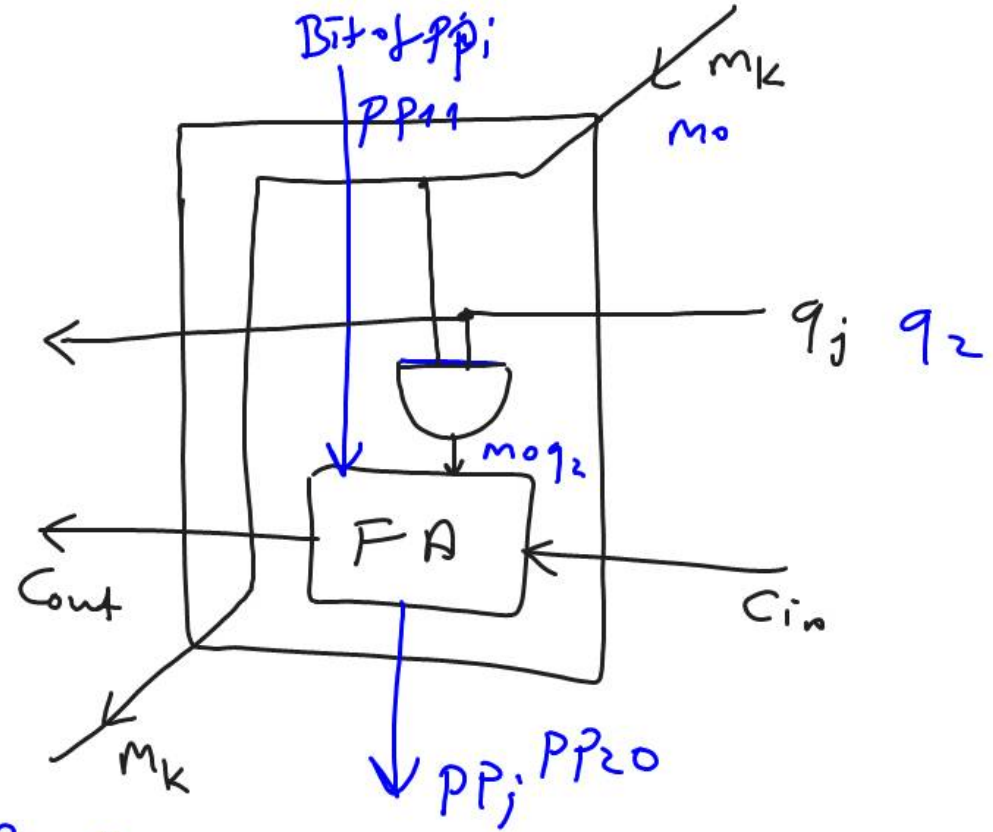
$$PP1 \quad 0 \quad PP1_3 \quad PP1_2 \quad PP1_1 \quad PP1_0 \quad PP1_0$$

$$+ m_3 q_2 \quad m_2 q_2 \quad m_1 q_2 \quad m_0 q_2 \quad 0 \quad 0$$

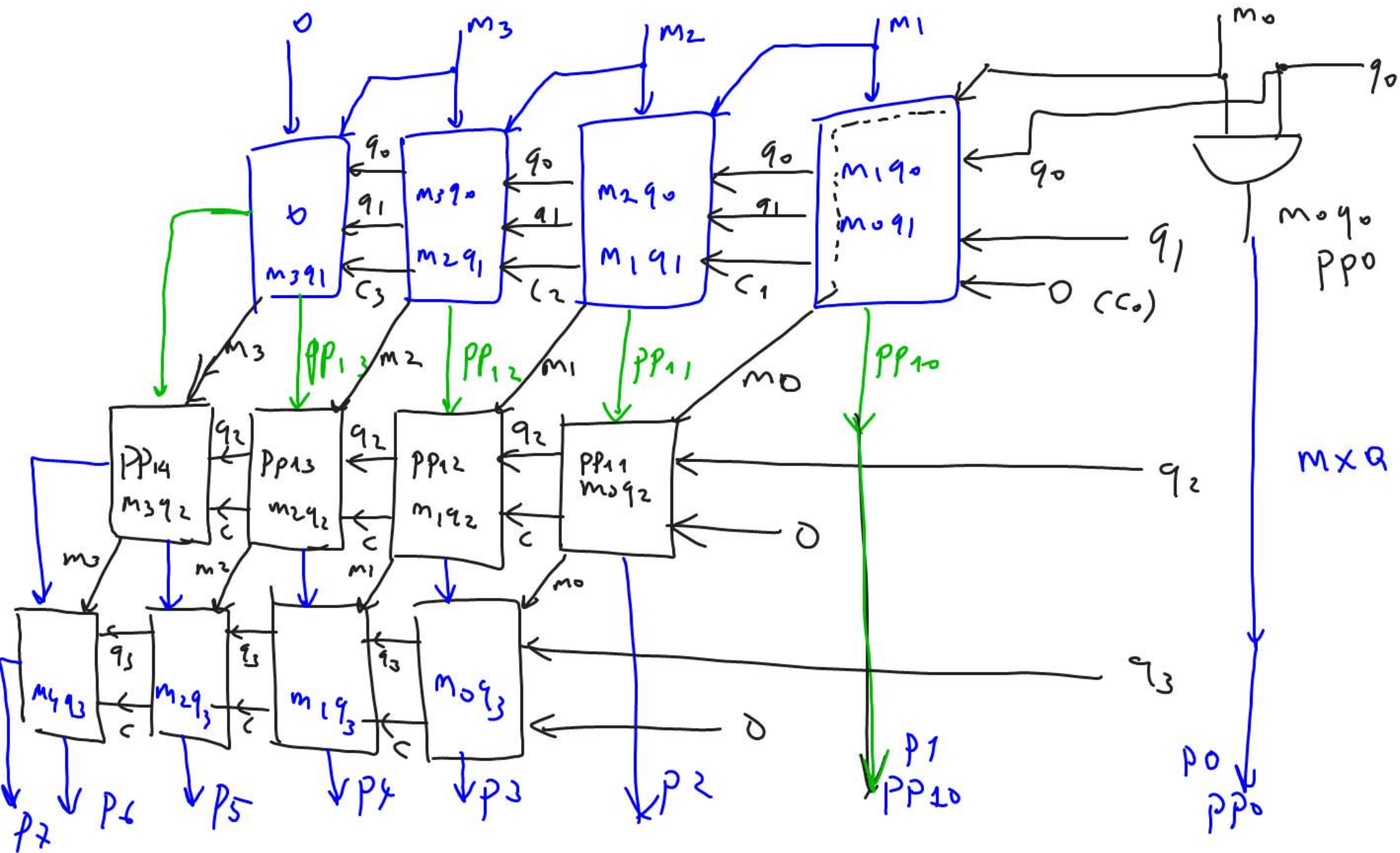
$$PP2 \quad PP2_3 \quad PP2_2 \quad PP2_1 \quad PP2_0 \quad \downarrow \quad \downarrow$$



1st Stage Building Block

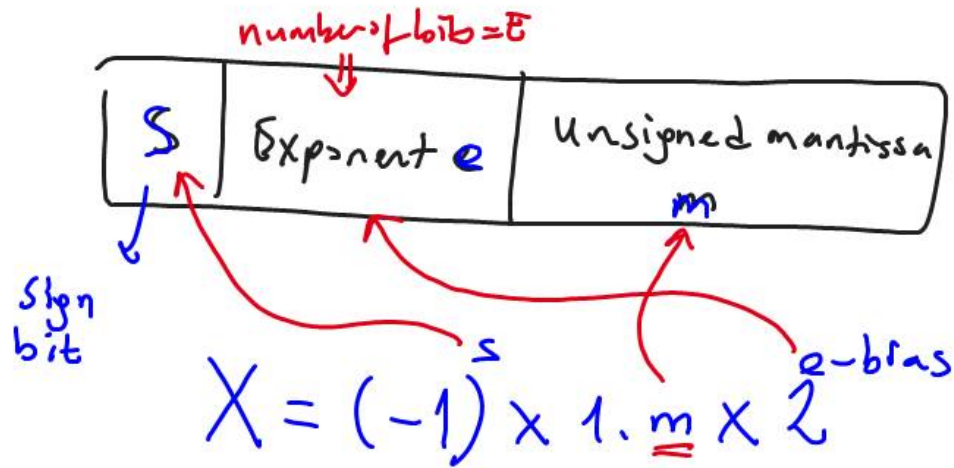


Building block for one other stages ( $i, j, \dots$ )



# FLOATING-POINT NUMBERS X FIXED-POINT NUMBERS

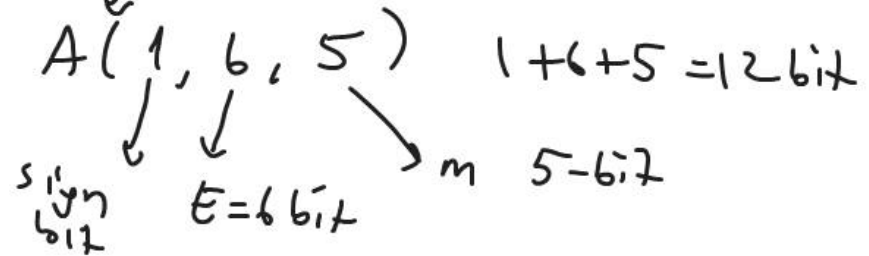
A floating point number:



$$\text{bias} = 2^{E-1} - 1$$

## Example:

Determine the decimal value 9.25 in a 12-bit Custom floating-point format



$$\text{bias} = 2^{E-1} - 1 = 2^5 - 1 = 31$$



$$9.25_{10} = 1001.01_2 = 1.00101 \times 2^3$$

$2^3$     $2^2$     $2^1$     $2^0$     $2^{-1}$     $2^{-2}$   
 $8+0+0+1=9$     $0+\frac{1}{4}=0.25$

$$X = (-1)^s \times 1.m \times 2^{e-\text{bias}}$$

$$m = \underline{00101}$$

5-bit

$$\underline{X = (-1)^0 \times 1.00101 \times 2^3}$$

$= 9.25_{10}$

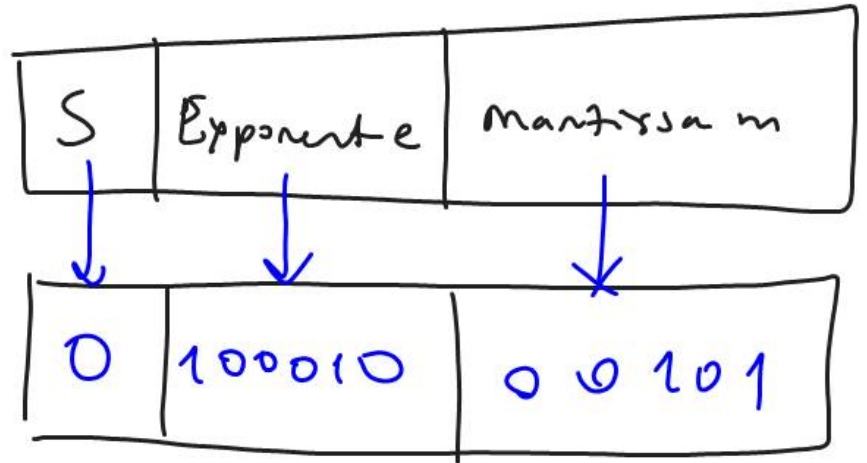
$$2^{e-\text{bias}} = 2^3$$

$$e = 3 + \text{bias}$$

$$e = 3 + 31 = 34_{10} = \underline{100010}_2$$

6-bit

$$9.25_{10} =$$



example back conversion

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COMBINATIONAL CIRCUITS

→ BCD adder

→ Decoder

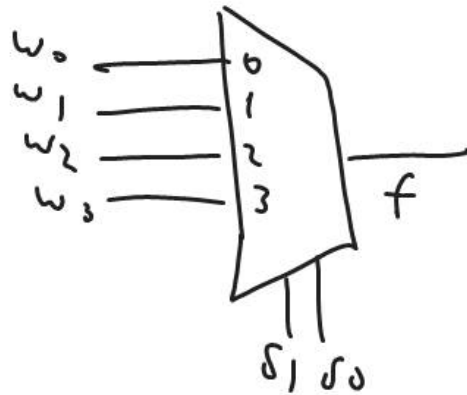
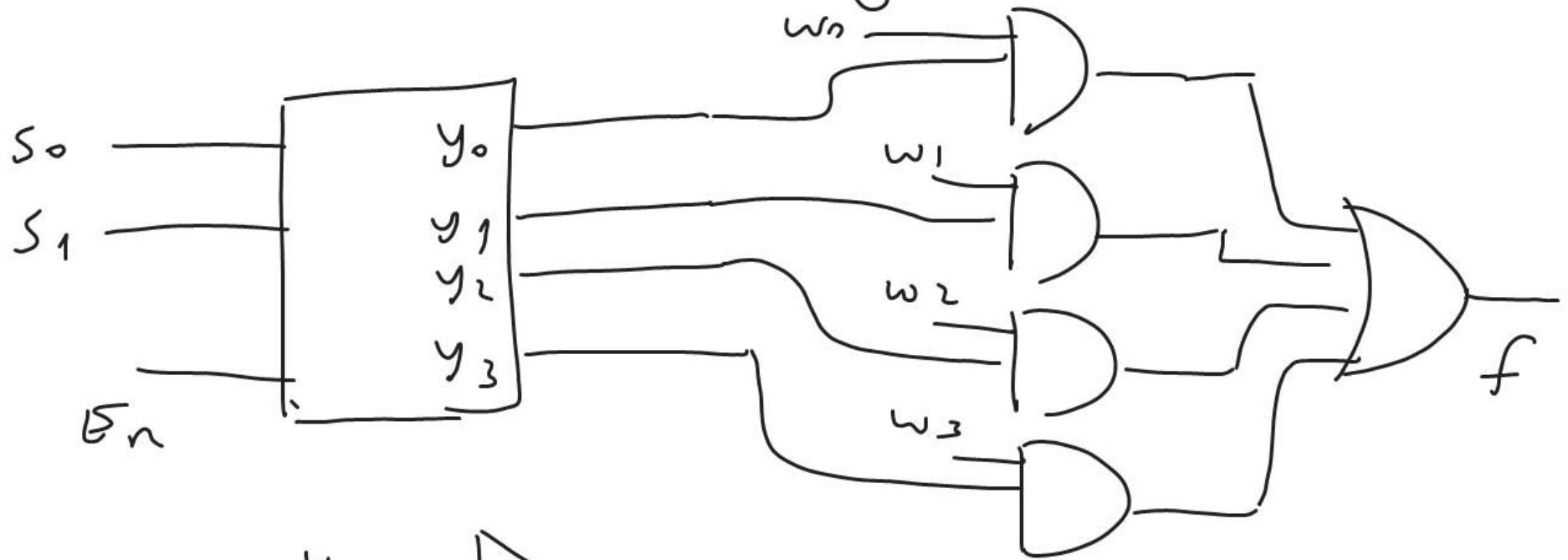
→

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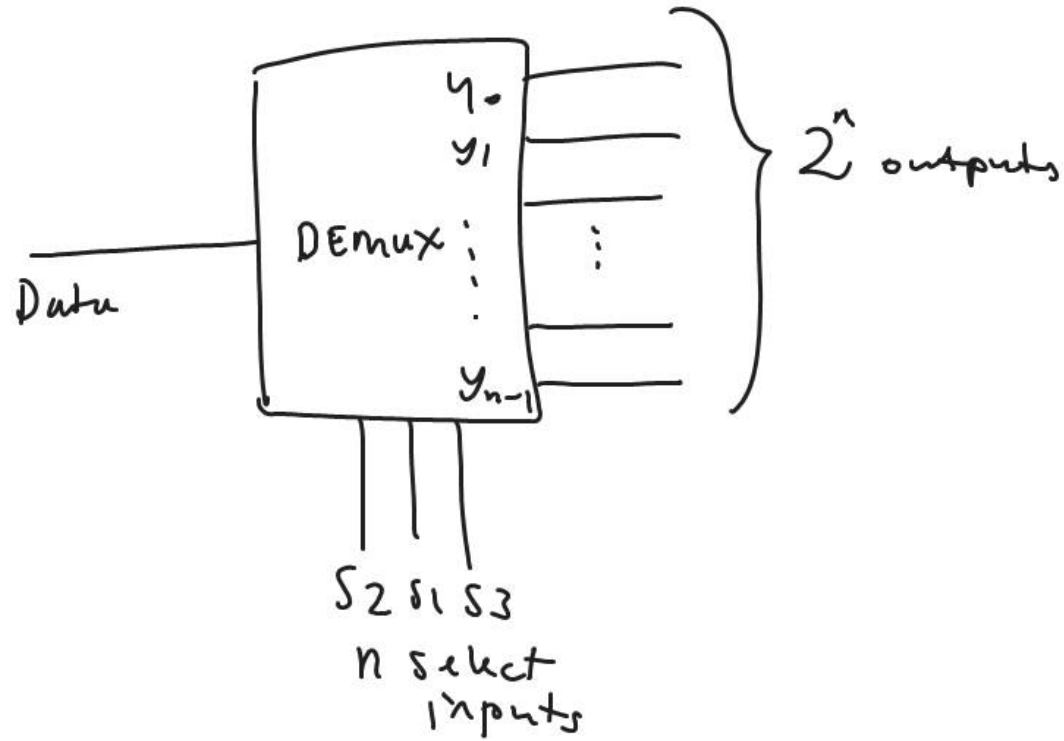
18.03.2010

2 hours

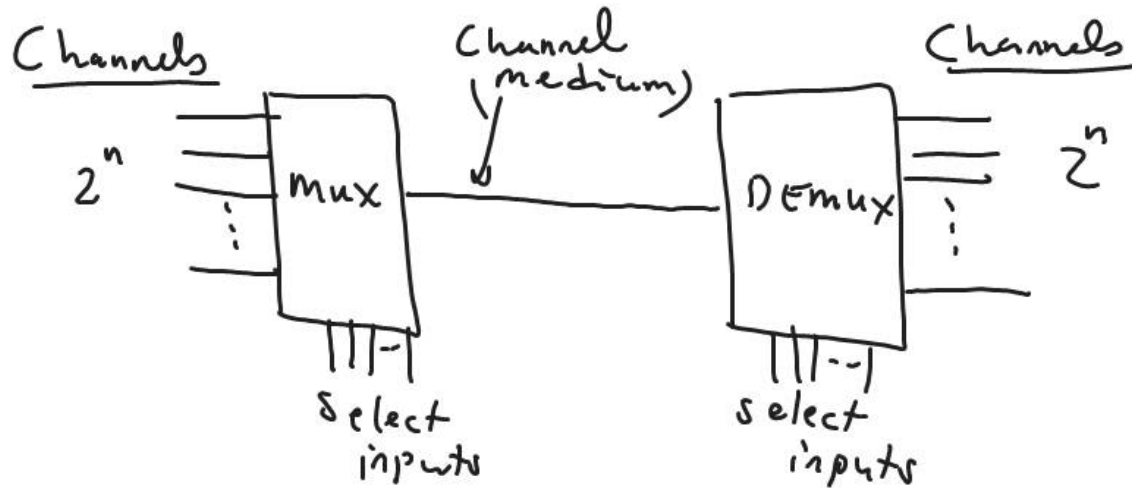
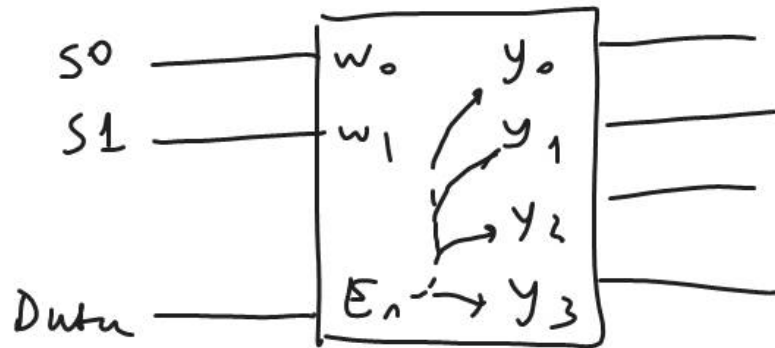
A 4-to-1 Mux using a decoder

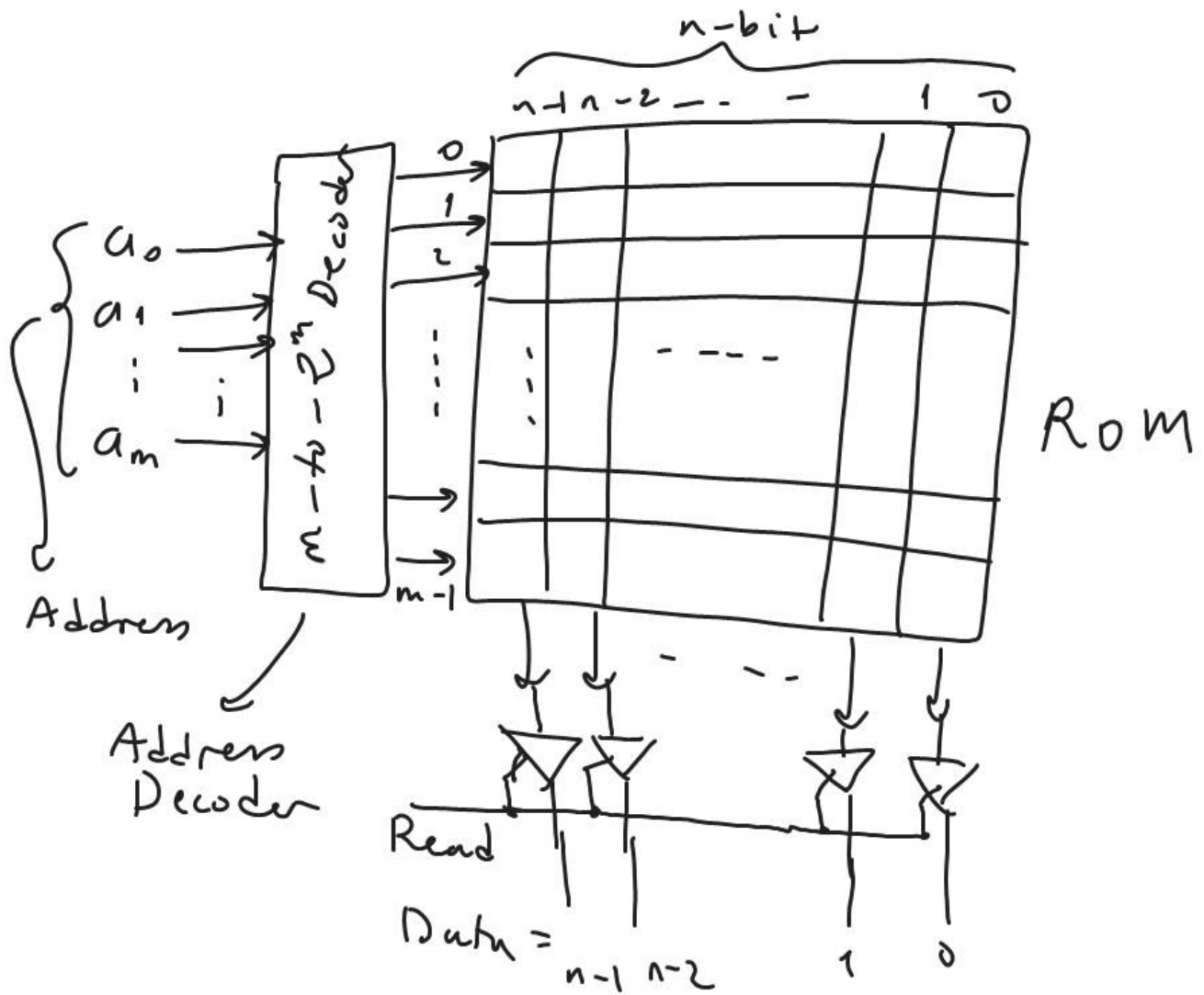


# DEMULTIPLEXERS



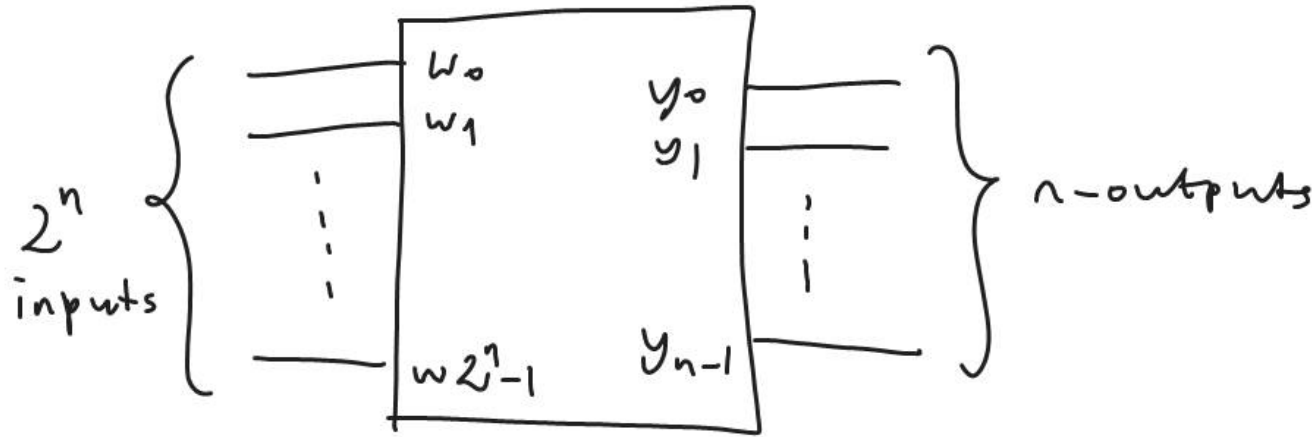
USING the DECODERS as DEMUXs:





A  $2^m \times n$  ROM block

# ENCODERS



(opposite of decoder)

A 4-to-2 Encoder

$w_3$	$w_2$	$w_1$	$w_0$	$y_1$	$y_0$
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1
0	0	0	0	N/A	N/A

→ Compression

→ using less memory  
location

# Priority Encoders:

Intermediate signals:

$$\bar{i}_0 = \bar{w}_3 \bar{w}_2 \bar{w}_1 w_0$$

$$\bar{i}_1 = \bar{w}_3 \bar{w}_2 w_1$$

$$\bar{i}_2 = \bar{w}_3 w_2$$

$$\bar{i}_3 = w_3$$

	$w_3$	$w_2$	$w_1$	$w_0$	$y_1$	$y_0$	$z$
	0	0	0	0	↓	↓	0
$i_0$	0	0	0	1	0	0	1
$i_1$	0	0	1	X	0	1	1
$i_2$	0	1	X	X	1	0	1
$i_3$	1	X	X	X	1	1	1

X → irrelevant (anything)  
if possible

↓ → don't care

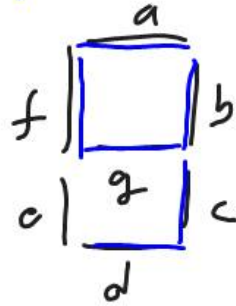
$$y_0 = \bar{i}_1 + \bar{i}_3$$

$$y_1 = \bar{i}_2 + \bar{i}_3$$

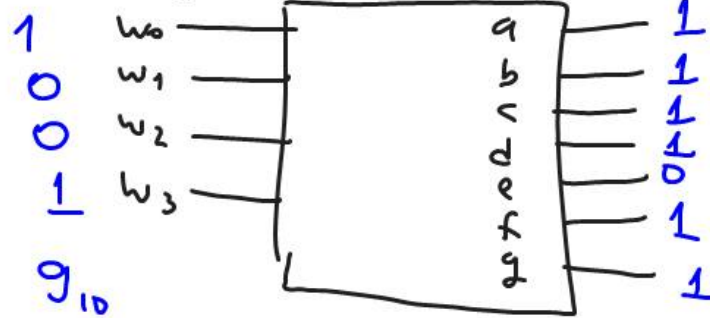
$$z = \bar{i}_0 + \bar{i}_1 + \bar{i}_2 + \bar{i}_3$$



Q BCD to 7-segment ~~encoder~~  
decoder } ?



a BCD digit



# ARITHMETIC COMPARISON CIRCUITS

$A, B$ , two unsigned numbers of  $n$ -bit each

The comparator outputs:

$$A \text{ eq } B \quad (A=B)$$

$$A \text{ gt } B \quad (A>B)$$

$$A \text{ lt } B \quad (A<B)$$

ex:  $n=4$

$$A = a_3 a_2 a_1 a_0$$

$$B = b_3 b_2 b_1 b_0$$

intermediate signals:

$$\bar{v}_3, \bar{v}_2, \bar{v}_1, i_0$$

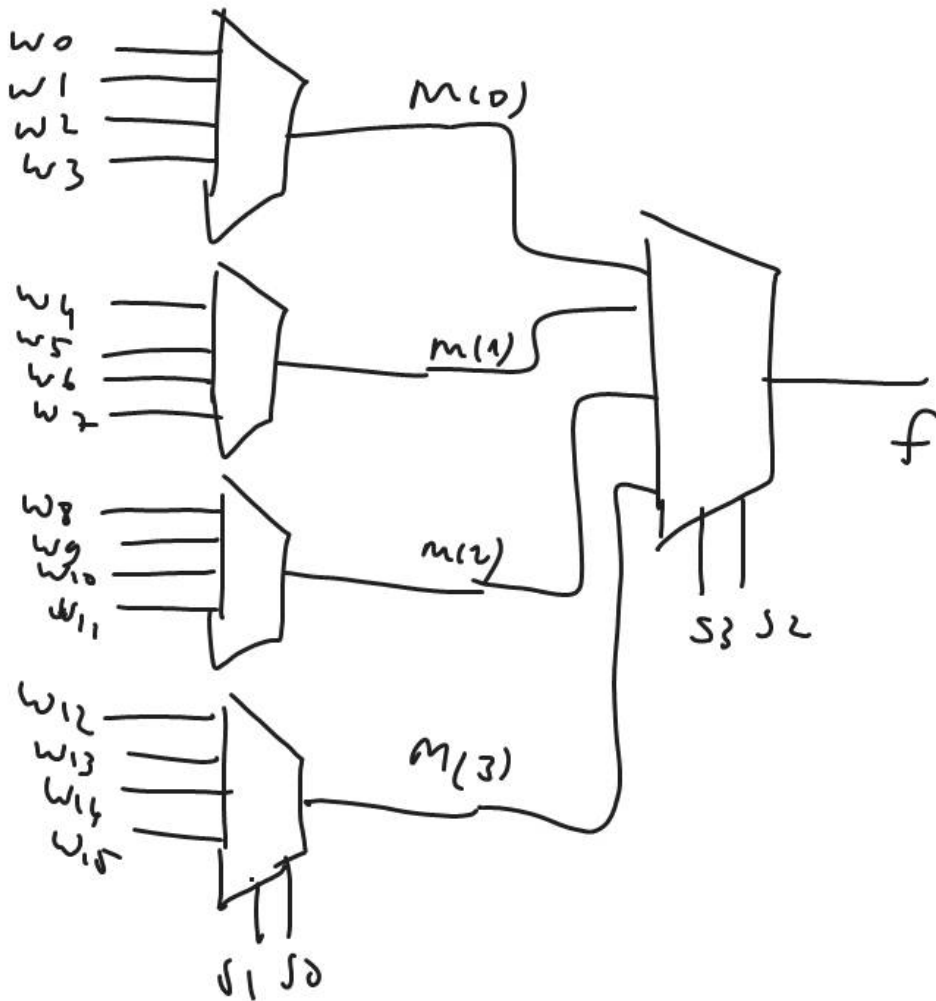
$$A=B \text{ when } a_k = b_k \text{ for } \begin{matrix} i=0 \\ \text{to} \\ n \end{matrix}$$

31 March 2010 ©

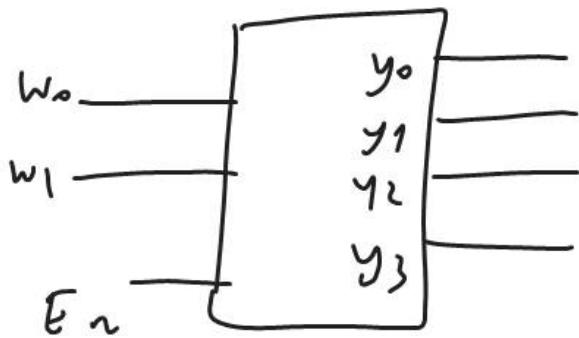
25 Mar 2010 (2 hours)

- Arithmetic comparison circuits
- Assignment statements
- 2 Selected
- → 4-to-1 mux | 2-to-1 mux

An 16-to-1 mux with 5 4-to-1 mux.



# 2-to-4 Binary Decoder



$E_n$	$w_1$	$w_0$	$y_0$	$y_1$	$y_2$	$y_3$
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1
0	X	X	0	0	0	0

ENTITY dec2to4 IS  
 PORT (w: IN STD\_LOGIC\_VECTOR  
 (1 DOWN TO 0),

En: IN STD\_LOGIC;

y: OUT STD\_LOGIC\_VECTOR  
 (1 TO 3));

END dec2to4;

ARCHITECTURE Behavior of dec2to4 IS

SIGNAL Enw = STD\_LOGIC\_VECTOR  
 (2 DOWN TO 0);

BEGIN

Enw <= En & w;

d: don't care

X = irrelevant  
 (not applicable)

WITH Enw SELECT

y <= "1000" WHEN "100";  
 "0100" WHEN "001";  
 "0010" WHEN "110";  
 "0001" WHEN "111";  
 "0000" WHEN OTHERS;

End Behavior;

## CONDITIONAL SIGNAL ASSIGNMENT

ENTITY mux2to1 IS

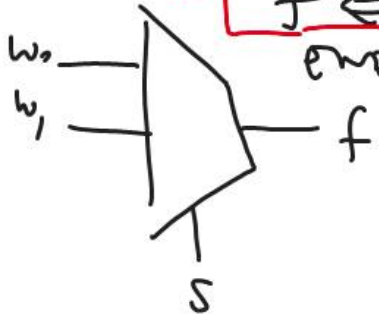
PORT (w0, w1, s : IN STD\_LOGIC;  
f : OUT STD\_LOGIC);

END mux2to1;

ARCHITECTURE Behavior OF 2-to-1 mux IS  
BEGIN

$f \leftarrow w_0$  WHEN  $s = '0'$  ELSE  $w_1$ ;

END Behavior;



2-to-1 mux

Both concurrent assignment  
statements.

Recall: with SELECT

ARCHITECTURE -----  
BEGIN

WITH s SELECT

$f \leftarrow w_0$  WHEN '0';  
 $w_1$  WHEN OTHERS;

END Behavior;

# A 4-to-1 PRIORITY ENCODER ENTITY Priority LS

	$w_3$	$w_2$	$w_1$	$w_0$	$y_1$	$y_0$	$z$
	0	0	0	0	d	d	0
$i_0$	0	0	0	1	0	0	1
$i_1$	0	0	1	X	0	1	1
$i_2$	0	1	X	X	1	0	1
$i_3$	1	X	X	X	1	1	1

```

PORT (w : IN STD_LOGIC_VECTOR (3 DOWN TO 0);
      y : OUT STD_LOGIC_VECTOR (1 DOWN TO 0);
      z : OUT STD_LOGIC);

```

END Priority;

ARCHITECTURE Behavior OF Priority LS

BEGIN

```

y <= "11" WHEN w(3)='1' ELSE
      "10" WHEN w(2)='1' ELSE
      "01" WHEN w(1)='1' ELSE
      "00";
z <= '0' WHEN w="0000" ELSE '1';
END Behavior;

```

CONDITIONAL Statement

$$y_1 = i_2 + i_3$$

$$y_0 = i_1 + i_3$$

$$z = i_0 + i_1 + i_2 + i_3$$

X = irrelevant  
d = don't care

$$\bar{i}_0 = \bar{w}_3 \bar{w}_2 \bar{w}_1 w_0$$

$$\bar{i}_1 = \bar{w}_3 \bar{w}_2 w_1$$

$$\bar{i}_2 = \bar{w}_3 w_2$$

$$\bar{i}_3 = w_3$$

# Priority encoder with SELECT STATEMENT

ARCHITECTURE -----

BEGIN

WITH W SELECT

y ← "00" WHEN "0001" ;

"01" WHEN "0010" ;

"01" WHEN "0011" ;

"10" WHEN "0100" ;

"10" WHEN "0101" ;

"10" WHEN "0110" ;

"10" WHEN "0111" ;

"11" WHEN OTHERS ;

WITH W SELECT

z ← '0' WHEN "0000"

'1' WHEN OTHERS ;

END Behavior ;



01 April 2010 2 hours

- Implementations of Comparators
  - GENERATE statement
    - For next (FOR GENERATE)
    - Windowed (||= GENERATE)
  - SEQUENTIAL ASSIGNMENT STATEMENTS
    - If then ELSE
    - CASE Statement
    - PROCESS Statement
- Ex: priority encoder

+ 5-9 April  
2010  
Exam Week

14 April 2010  
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# Priority Encoder with IF-THEN

## ARCHITECTURE Behavior of Priority

```
BEGIN
```

```
PROCESS(w)
```

```
BEGIN
```

```
  y ← "00";
```

```
  IF w(1) = '1' THEN y ← "01"; END IF;
```

```
  IF w(2) = '1' THEN y ← "10"; END IF;
```

```
  IF w(3) = '1' THEN y ← "11"; END IF;
```

```
  z ← '1';
```

```
  IF w = "0000" THEN z ← '0'; END IF;
```

Within one PROCESS the values assigned to the parameters may change. But the latest value is the valid one.

```
END PROCESS;
```

```
END BEHAVIOR;
```

## Example: Comparator

ENTITY Comparator IS

```
PORT ( A, B : IN STD_LOGIC;  
       AeqB : OUT " " );
```

END Comparator

ARCHITECTURE Behavior OF Comparator IS

BEGIN

PROCESS (A, B)

BEGIN

AeqB <= '0';

IF A = B THEN

AeqB <= '1';

ENDIF;

END PROCESS;  
END Behavior;

THE DEFAULT  
ASSIGNMENT STATEMENT

# IMPLIED MEMORY

ENTITY Implied IS

```
PORT (A, B: IN STD_LOGIC;
      A eq B: OUT " ");
```

END Implied;

ARCHITECTURE Behavior OF Implied IS

BEGIN

PROCESS (A, B)

BEGIN

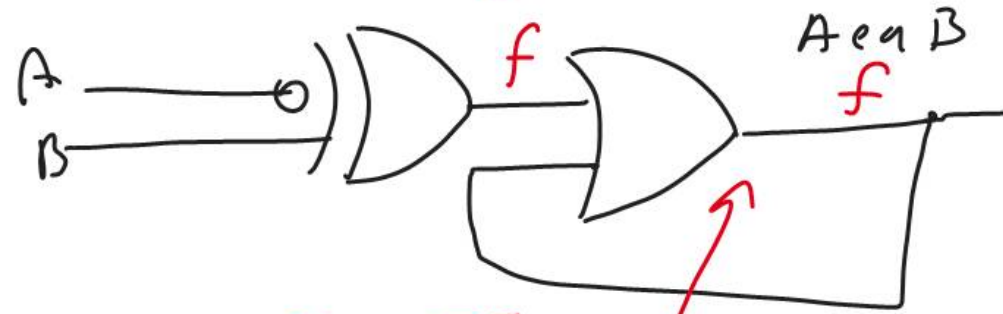
IF A = B THEN

A eq B <= '1';

END IF;

END PROCESS;

END Behavior;



$$f = \bar{A} \oplus B = \bar{\bar{A}}B + \bar{A}\bar{B}$$

$$f = AB + \bar{A}\bar{B}$$

A	B	f
0	0	1
0	1	0
1	0	0
1	1	1

IMPLIED MEMORY!!  
MISTAKE/ERROR

Value f will be kept forever!

# CASE STATEMENT

ex an 2-to-1 mux

ARCHITECTURE Behavior of mux<sup>2</sup> is

BEGIN

PROCESS (w1, w0, s)

BEGIN

CASE s IS

WITH '0' =>

f <= w0;

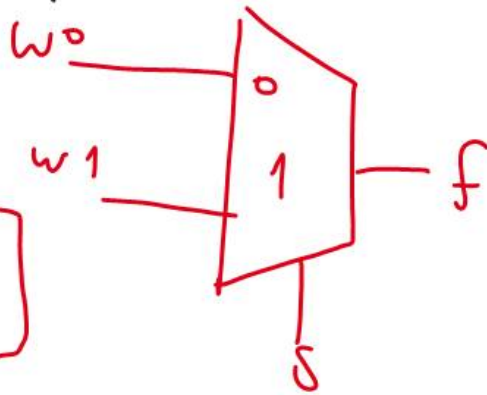
WITH OTHERS =>

f <= w1;

END CASE;

END PROCESS;

END Behavior;



## RECALL:

① CONDITIONAL

f <= w0 WHEN s = '0'  
ELSE w1;

② WITH s SELECT

f <= w0 WITH '0';

w1 WITH OTHERS;

③ IF-THEN-ELSE

PROCESS (w0, w1, s)

BEGIN

IF s = '0' THEN f <= w0;

ELSE f <= w1;

END IF;

END PROCESS;