sign bit, n=4	1's complement	2's complement
0 000	0	0
0 001	1	1
0 010	2	2
0 011	3	3
0 100	4	4
0 101	5	5
0 110	6	6
0 111	7	7
1 000	−7	−8
1 001	−6	−7
1 010	−5	−6
1 011	−4	−5
1 100	−3	−4
1 101	−2	−3
1 110	−1	−2
1 111	−0	−1

positive numbers: 0000–0111

negative numbers: 1000–1111

1's complement:
- 2 zeroes
- $2^{n-1}-1$ positive
- $2^{n-1}-1$ negative numbers

2's complement:
- 1 zero
- $2^{n-1}-1$ positive
- 2^{n-1} negative numbers

07-03.2011

©

CZC

CZ)

1's complement addition:

$$\begin{array}{r} +5 \\ + \; +2 \\ \hline +7 \end{array} \qquad \begin{array}{r} -5 \\ + \; +2 \\ \hline -3 \end{array} \qquad \begin{array}{r} +5 \\ + \; -2 \\ \hline +3 \end{array} \qquad \begin{array}{r} -5 \\ + \; -2 \\ \hline -7 \end{array}$$

$$\begin{array}{r} 0101 \\ + \; 0010 \\ \hline 0111 \end{array} \;\checkmark$$

$$\begin{array}{r} 1010 \\ + \; 0010 \\ \hline 1100 \end{array} \qquad -3 \;\checkmark$$

$$\begin{array}{r} 0101 \\ + \; 1101 \\ \hline 1\,|\,0010 \\ 1 \\ + \quad\quad \\ \hline 0011 \end{array}$$

Carry is added to the LSB

$$\begin{array}{r} 1010 \\ + \; 1101 \\ \hline 1\,|\,0111 \\ 1 \\ + \quad\quad \\ \hline 1000 \end{array} \qquad -7 \;\checkmark$$

2's complement addition

+5
+2
+
7

0101
0010
+
0111
✓

0010
1
0011

−5
+2
+
−3

1011
0010
+
1101
−3 ✓

0010
1
0011

+5
−2
+
3

0101
1110
+
$\boxed{1}$ 0011
~~−~~+3 ✓

~~11101~~
1101
1
1110
−2

Carry is ignored.

−5
−2
+
−7

1011
1110
+
$\boxed{1}$ 1001 ✓
−7

1010
1
1011

0101
~~0101~~
0110
−5

SUBTRACTION

$$\begin{array}{r} 5 \\ -\ 2 \\ \hline \end{array} \Rightarrow \begin{array}{r} +5 \\ -2 \\ + \quad \\ \hline \end{array}$$

OVERFLOW

example:

$$\begin{array}{r} +7 \\ +\ +3 \\ \hline +10 \end{array} \qquad \begin{array}{r} 0111 \\ +\ 0011 \\ \hline 1010 \end{array} \qquad n = 4$$

OVERFLOW problem here!

$$\begin{array}{r} 0101 \\ 1 \\ \hline 0110 \end{array} \quad \underline{\underline{-6}} \;\times$$

$$\begin{array}{r} -6 \\ -5 \\ \hline + \quad -11 \end{array} \qquad \begin{array}{r} 1010 \\ 1011 \\ \hline + \quad 1\underbrace{0101}_{n=4} \end{array} \quad -7$$

overflow problem here.

$$\begin{array}{r} 1001 \\ 1 \\ \hline 1110 \end{array} \qquad \begin{array}{r} 0110 \\ 1 \\ \hline 1 \end{array} \qquad 0110 \rightarrow \begin{array}{r} 1001 \\ 1 \\ \hline 1010 \end{array}$$

$$\begin{array}{r} 1010 \\ 1 \\ \hline \end{array}$$

* If numbers of different SIGNS are added NO OVERFLOW occurs!

* if numbers of <u>the same SIGNS</u> are added OVERFLOW may occur!

* When there is an OVERFLOW we should use 1 more bit to properly represent the result.

$$
\begin{array}{r}
1010 \\
+\ 1011 \\
\hline
1\ \underbrace{0101}
\end{array}
$$

n=4 bit

+5 X

−7

OVERFLOW PROBLEM!

If we use n=5

$$
\begin{array}{rr}
-6 = 00110 & 5\text{bit} \\
11001 & \\
1 & \\
\hline
11010 & -6
\end{array}
$$

$$
\begin{array}{rr}
-5 = 00101 & \\
11010 & \\
1 & \\
\hline
11011 & -5
\end{array}
$$

$$
\begin{array}{rr}
11010 & -6 \\
+\ 11011 & -5 \\
\hline
1\ 10101 & \Rightarrow -11
\end{array}
$$

$$
\begin{array}{r}
01010 \\
1 \\
\hline
01011
\end{array}
$$

ex

$$\begin{array}{r} +5 \\ +\cancel{3}4 \\ \hline +9 \end{array}$$

$n=4$

$$\begin{array}{r} 0101 \\ +\ 0100 \\ \hline 1001 \end{array} \quad -7 \quad \rightarrow$$

WRONG!

overflow!

$$\begin{array}{r} 0110 \\ 1 \\ \hline 0111 \end{array}$$

$n=5$

$5 \Rightarrow 00101$

$4 \Rightarrow 00100$

$$\begin{array}{r} 00101 \\ +\ 00100 \\ \hline 01001 \end{array}$$

✓

+9 in 5 bit

Detection of Overflow:

① The numbers added should be of the same sign!

② use the carry bits to detect the overflow

if $\underline{C_{i+1} \oplus C_i = 1}$ then there is overflow!

ex

$$\begin{array}{r} +5 \\ +4 \\ \hline +9 \end{array} \qquad n=4 \qquad \begin{array}{rcccc} i= & 3 & 2 & 1 & 0 \\ & 0 & 1 & 0 & 1 \\ + & 0 & 1 & 0 & 0 \\ \hline & 1 & 0 & 0 & 1 \end{array} \leftarrow C_0$$

-7 ✗

$C_4 \quad C_3 \quad C_2 \quad C_1$

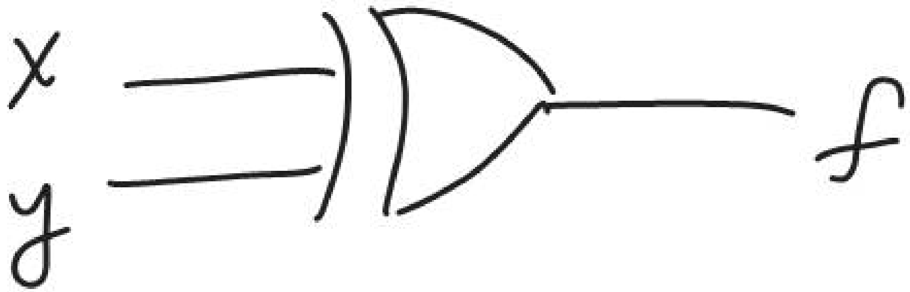
$C_{i+1} \quad C_i$

$C_4 = 0$

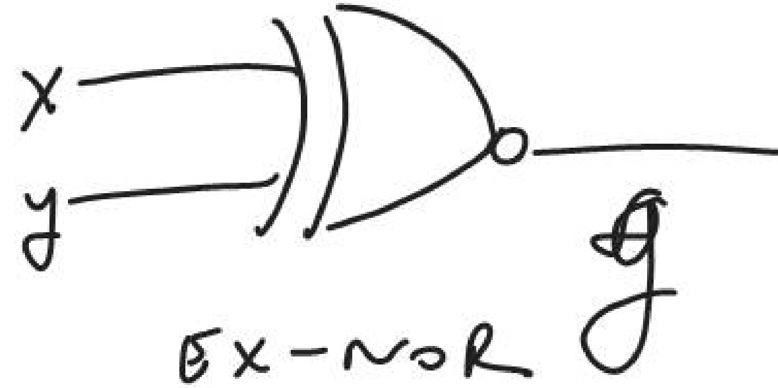
$C_3 = 1$

$C_4 \oplus C_3 = ?$

$0 \oplus 1 = 1$ ✓



ex-or



EX-NOR

x	y	
0	0	0
0	1	1
1	0	1
1	1	0

either but not both

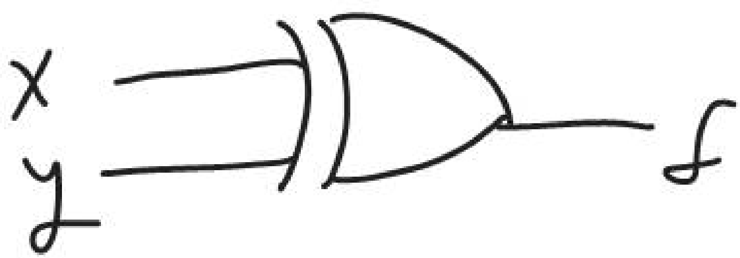
$x \oplus y = f$

x	y	g
0	0	1
0	1	0
1	0	0
1	1	1

Both but not either

$x \odot y = g$

$\overline{x \oplus y} = x \odot y$



$x = 0$

$f = y$

$x = 1$

$f = \overline{y}$

x	y	
0	0	0
0	1	1
1	0	1
1	1	0