Arithmetic Comparison Circuits

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A, B are two unsigned binary numbers.

Comparator outputs:
- A eq B $(A = B)$
- A gt B $(A > B)$
- A lt B $(A < B)$

<u>ex</u> $n = 4$

$A = a_3 a_2 a_1 a_0$

$B = b_3 b_2 b_1 b_0$

intermediate signals:

$i_3 = \overline{a_3 \oplus b_3}$

$i_2 = \overline{a_2 \oplus b_2}$ if $i_3 \cdot i_2 \cdot i_1 \cdot i_0 = 1$

$i_1 = \overline{a_1 \oplus b_1}$ $A = B$

$i_0 = \overline{a_0 \oplus b_1}$

$A > B$

$$a_3\overline{b_3} + i_3 a_2\overline{b_2} + a_1\overline{b_1} + a_0\overline{b_0} = AgtB$$

$i_3 \cdot i_2$ $\qquad$ $i_3 \cdot i_2 \cdot i_1$

Check if the previous bits are equal.

$A = 1011$

$B = 0101$

$a_3 \cdot \overline{b_3} = 1$

$$\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}$$

AgtB

$$= a_3\overline{b_3} + i_3 \cdot a_2 \cdot \overline{b_2} + i_3 \cdot i_2 \cdot a_1 \cdot \overline{b_1} + i_3 \cdot i_2 \cdot i_1 \cdot a_0 \cdot \overline{b_0}$$

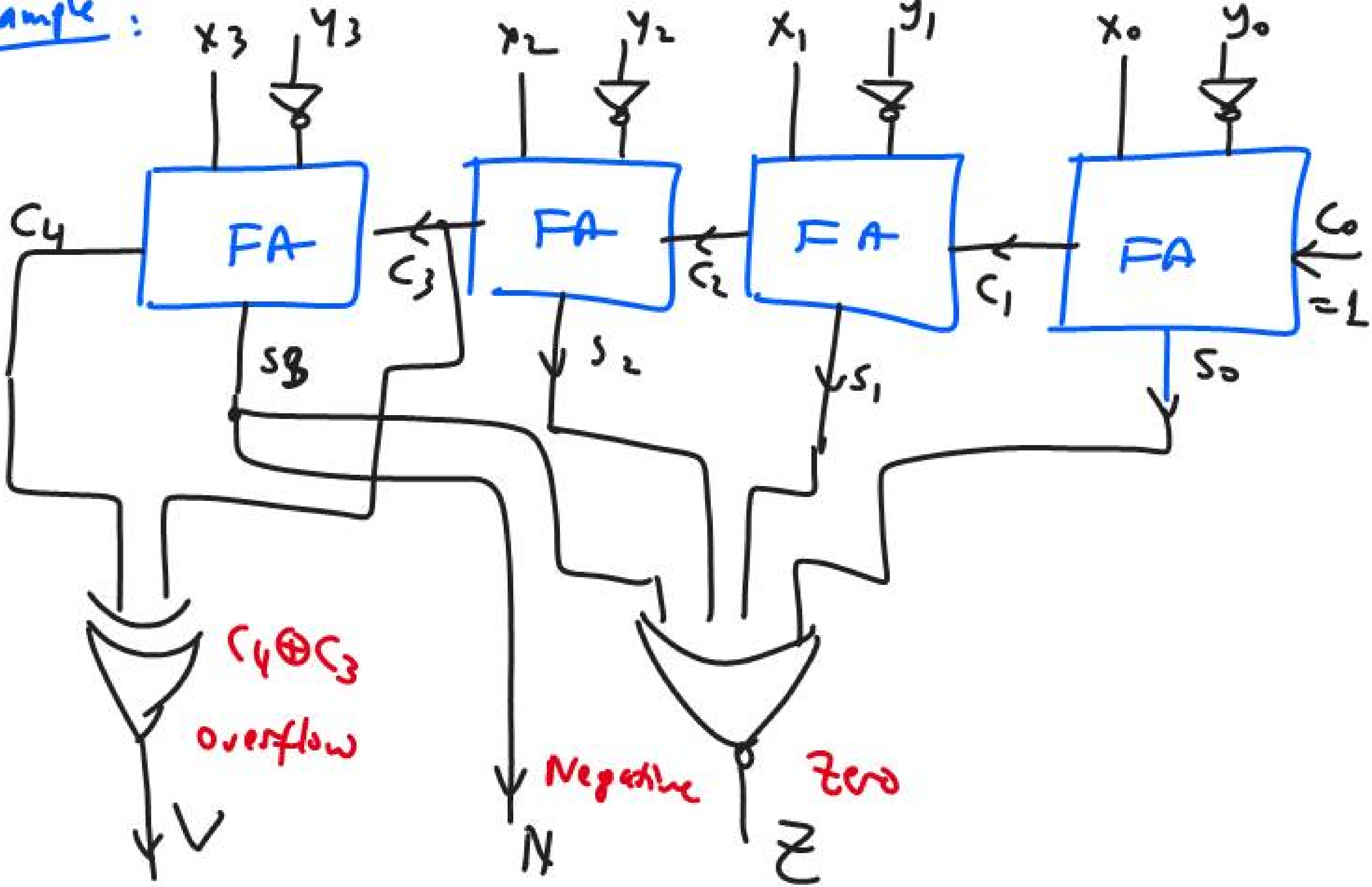
AltB:

$$\overline{a_3}b_3 + i_3 \cdot \overline{a_2}b_2 + i_3 \cdot i_2 \cdot \overline{a_1}b_1 + i_3 \cdot i_2 \cdot i_1 \cdot a_0\overline{b_0}$$

or $AltB = \overline{AeqB + AgtB}$

making a 2's complement subtraction (X−Y):

example:



STRUCTURAL VHDL CODE:

```
LIBRARY ieee;
USE ieee.std_logic_1164.all;
USE work.fulladd_package.all;
ENTITY COMPARATOR IS
PORT( X,Y :IN STD_LOGIC_VECTOR(3 DOWNTO 0);
        V,N,Z : OUT STD_LOGIC );
END COMPARATOR;
ARCHITECTURE Structure OF COMPARATOR IS
SIGNAL S : STD_LOGIC_VECTOR(3 DOWNTO 0);
SIGNAL C : STD_LOGIC_VECTOR (1 TO 4);
```

Co is always '1'

BEGIN

```
Stage 0: fulladd  PORTMAP ('1', X(0), NOT Y(0), S(0), C(1));
Stage 1:    "         "   (C(1), X(1), NOT Y(1), S(1), C(2));
Stage 2:    "         "   (C(2), X(2), NOT Y(2), S(2), C(3));
Stage 3:    "         "   (C(3), X(3), NOT Y(3), S(3), C(4));

V <= C(4) EXOR C(3);

Z <= '1' WHEN S(3 DOWNTO 0) = "0000" ELSE '0';

N <= S(3);

END STRUCTURE;
```

① If $X < Y \Rightarrow X - Y$ operation we are doing

a) X and Y are of the same sign

$$\begin{array}{r} X \\ -\ Y \\ \hline \end{array} \Rightarrow \begin{array}{r} X \\ +\ -Y \\ \hline \end{array}$$

$V=0$ No overflow

$N=1$ result is always negative ✓

①

b) X and Y are of the opposite signs.

$x=-5$
$y=+4$

$-5-(+4)=-9$

X is negative
Y is positive

$X - Y =$

$-2 \quad 3 \quad -2-(3)=-5$

$$\begin{array}{r} 0101 \\ 1010 \\ 1 \\ \hline 1011 \\ -5 \end{array} \qquad \begin{array}{r} 0100 \\ 1000 \\ 1 \\ \hline 1001 \\ -4 \end{array} \qquad \begin{array}{r} 1011 \\ +1001 \\ \hline 10100 \end{array}$$

X is positive
Y is negative ~~X (X<Y)~~

$\left.\begin{array}{l} N=1 \\ V=0 \end{array}\right\}$ If there is no overflow.

②

If there is overflow $\left.\begin{array}{l} V=1 \\ N=0 \end{array}\right\}$ ③

$\therefore$ we can write $N \oplus V = 1$ for $X < Y$. in $X - Y$.

* $X = Y \Rightarrow z = 1$
* $X \leq Y \Rightarrow z + N \oplus V = 1$
* $X > Y = \overline{(X \leq Y)} \Rightarrow \overline{(z + N \oplus V)} = 1$
* $X \geq Y = z + \overline{(z + N \oplus V)} = 1$

$$z + \bar{z} \cdot \overline{(N \oplus V)} = 1$$

$$\underbrace{(z + \bar{z})}_{1} \left(z + \overline{(N \oplus V)} \right) = 1$$

$= \overline{X < Y}$

$= \overline{N \oplus V} = 1$